**United Kingdom
Mathematics Trust**

Intermediate Mathematical Olympiad
Cayley paper
Thursday 21 March 2019
Organised by the United Kingdom Mathematics Trust

Supported by

6verleaf

England & Wales: Year 9 or below
Scotland: S2 or below
Northern Ireland: Year 10 or below

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

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◇ *Try to finish whole questions even if you cannot do many.*

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1. Each of Alice and Beatrice has their birthday on the same day.

In 8 years' time, Alice will be twice as old as Beatrice. Ten years ago, the sum of their ages was 21.

How old is Alice now?

2. In the addition shown, each of the letters D, O, G, C, A and T represents a different digit.

What is the value of $D + O + G + C + A + T$?

$$\begin{array}{r} D\,O\,G \\ +\,C\,A\,T \\ \hline 1\,0\,0\,0 \end{array}$$

3. The triangle ABC is isosceles with $AB = BC$. The point D is a point on BC, between B and C, so that $AC = AD = BD$.

What is the size of angle ABC?

4. Arrange the digits 1, 2, 3, 4, 5, 6, 7, 8 to form two 4-digit integers whose difference is as small as possible.

Explain clearly why your arrangement achieves the smallest possible difference.

5. Howard chooses n different numbers from the list 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, so that no two of his choices add up to a square.

What is the largest possible value of n?

6. A chessboard is formed from an 8×8 grid of alternating black and white squares, as shown. The side of each small square is 1 cm.
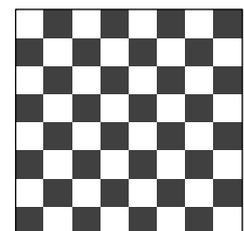
What is the largest possible radius of a circle that can be drawn on the board in such a way that the circumference is entirely on white squares or corners?